6d strings from new 2d chiral gauge theories

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Talk based on collaboration with

Hee-Cheol Kim, Jaemo Park, “6d SU(3) self-dual strings,” work in preparation.

& more works in progress.

Related background materials:

6d self-dual strings:

Haghighat, Iqbal, Kozcaz, Lockhart, Vafa, “M-strings” 1305.6322
Joonho Kim, SK, Kimyeong Lee, Jaemo Park, Vafa, “Elliptic genus of E-strings” 1411.2324
Haghighat, Klemm, Lockhart, Vafa, “Strings of 6d minimal SCFTs” 1412.3152
Gadde, Haghighat, Joonho Kim, SK, Lockhart, Vafa, “6d string chains” 1504.04614
Joonho Kim, SK, Kimyeong Lee, “Higgsing towards E-strings” 1510.03128

6d SCFTs from F-theory:

Morrison, Taylor, “Classifying bases for 6D F-theory models” 1201.1943
Heckman, Morrison, Vafa, “On the classification of 6D SCFTs … ” 1312.5746
Heckman, Morrison, Rudelius, Vafa, “Atomic classification of 6D SCFTs” 1502.05405

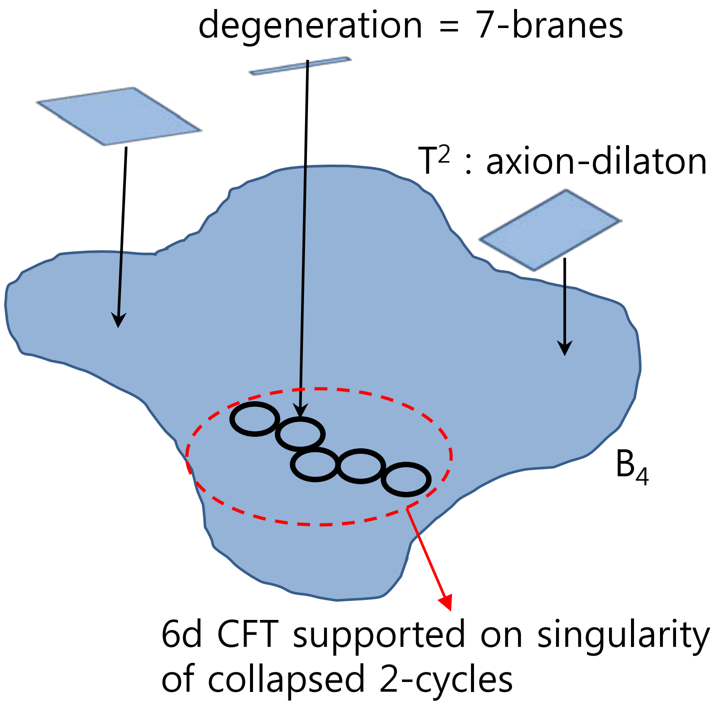
6d SCFTs from F-theory

- 6d SCFTs from F-theory on $R^{5,1}$ x (elliptic CY_3)
- "Atomic classification" by going to tensor branch

$$B_{\mu\nu} \text{ with } H = dB = \star dB \ , \quad \Psi^A \ , \quad \boxed{\Phi} \longrightarrow \text{VEV}$$

[Morrison, Taylor] (2012) [Heckman, Morrison, Vafa] (2013)

[Heckman, Morrison, Rudelius, Vafa] (2015)



- Key object in tensor branch: "self-dual strings"
- D3-branes wrapping 2-cycles
- Analogous to 4d W-bosons/monopoles/dyons in Coulomb branch

- half-BPS strings in 6d (1,0) theory: 2d N=(0,4) SCFTs on worldsheet
- Using 2d approach, one can microscopic study a subsector of 6d SCFT
- related to other problems (Yang-Mills instantons, compactification of 4d isolated SCFT)
- key ingredient to study some 6d CFT observables (e.g. superconformal index)

The “atoms”

- 6d minimal SCFTs [Witten] [Morrison,Vafa] [Haghighat, Klemm, Lockhart, Vafa] (1996)
- defined by two conditions: has 1 tensor multiplet & non-Higgsable gauge symmetry
- building blocks of complicated SCFTs

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

- Effective field theory in the tensor branch
- 6d SYM + hypermultiplets, coupled to tensor multiplets
- Severely restricted by consistency conditions (anomaly cancelation)

$$S^{\rm bos}_{\rm v+t} = \int \left[\frac{1}{2}d\Phi\wedge\star d\Phi + \frac{1}{2}H\wedge\star H\right] + \sqrt{c}\int\left[-\Phi{\rm tr}(F\wedge\star F) + B\wedge{\rm tr}(F\wedge F)\right]$$

$$H \equiv dB + \sqrt{c}\,{\rm tr}\left(AdA - \frac{2i}{3}A^3\right)$$

- To make more complicated 6d SCFTs from the minimal SCFTs,
- combine minimal SCFTs to make “quivers” (tensor scalar sets Yang-Mills coupling)
- “unHiggs” to bigger gauge groups w/ more hypermultiplet matters

6d super-Yang-Mills & soliton strings

- minimal SCFTs w/ gauge symmetry:

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

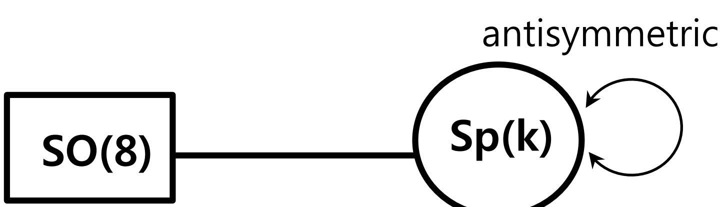
- Self-dual strings are soliton strings

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\,(F \wedge F) \in \mathbb{Z} \qquad S \leftarrow \int B \wedge \mathrm{tr}(F \wedge F)$$

- 4d instantons: represents vacuum tunneling
- 5d “instantons” = particles: important roles in 5d Yang-Mills descriptions of 5d/6d SCFTs
- 6d instanton strings: Their quantum dynamics is surprisingly ill-understood…

- Classical gauge group: ADHM construction suggests 2d gauge theories on these strings.

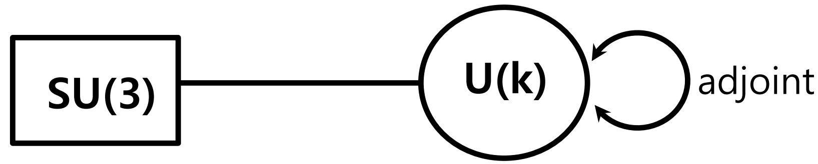
- Apparently simple cases: n=3,4 w/ SU(3), SO(8)
- n=4: [Haghighat, Klemm, Lockhart, Vafa]
- SO(8) ADHM: 2d Sp(k) gauge theory for k strings



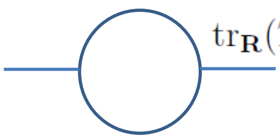
Strings of minimal SCFTs: n=3

- SU(3) is smallest non-Higgsable gauge group (non-Higgsable SU(2) is forbidden)
- Naively, one can guess from SU(3) ADHM consturction

- SU(3) ADHM for k instantons:



- U(k) anomaly doesn't cancel:



$\mathrm{tr}_{\mathbf{R}}(T^aT^b) = D_{\mathbf{R}}\delta^{ab}$

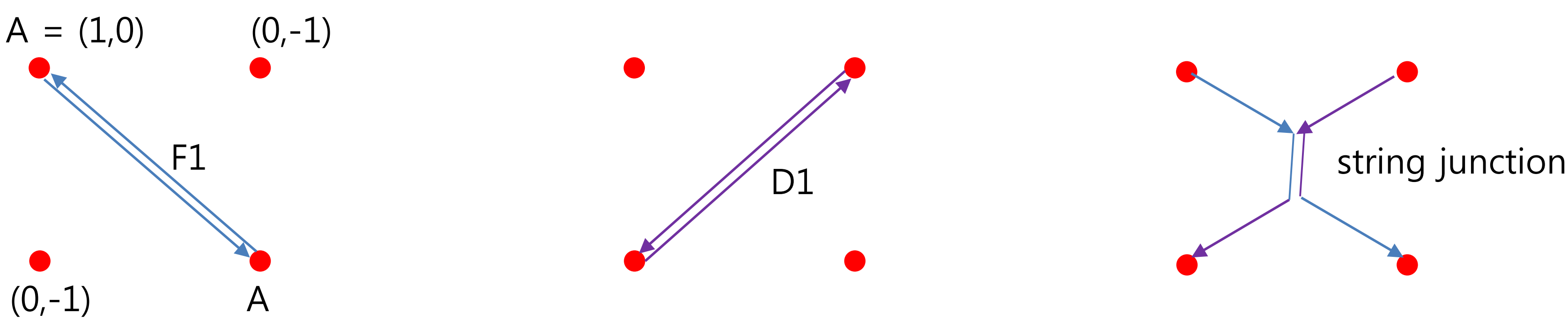
$D_{\mathbf{k}} = 1$

$D_{\mathbf{adj}} = 2k.$

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot\alpha A-}$	**adj**	**1**	**1**	**2**	**2**
$q_{\dot\alpha}(\to \psi_{A+})$	**k**	$\bar{\mathbf{3}}$	**1**	**2**	**1**
$a_{\alpha\dot\beta}(\to \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**

$\sim\ 2\cdot 3\cdot 1 + 2\cdot 2k - 2\cdot 2k = 6 \neq 0$

- The failure is "natural": "nonperturbative" "exceptional" SU(3) [Grassi,Halverson,Shaneson]



The cure for SU(3)

- Strategy: bottom-up approach, cure the pathology of naïve SU(3) quiver.
- Proposal: can't make N=(0,4) gauge theory uplift, but can do N=(0,2) uplift.

- Add the following N=(0,2) matters:

$$\begin{array}{rcl}
(\phi,\chi) & : & \text{chiral multiplet in } (\bar{\mathbf{k}},\bar{\mathbf{3}}) \\
(b_{1,2},\xi_{1,2}) & : & \text{two chiral multiplet in } (\overline{\mathbf{anti}},\mathbf{1}) \\
\hat{V}=(\hat{A}_-,\hat{\lambda},\bar{\mu},\hat{D}) & : & \text{complex vector multiplet in } (\mathbf{sym},\mathbf{1}) \\
(\check{\lambda},G_{\check{\lambda}}) & : & \text{complex Fermi multiplet in } (\mathbf{sym},\mathbf{1}) \\
v=(a_-,\zeta,\bar{\mu}_\zeta,D_\zeta) & : & \text{complex vector multiplet in } (\bar{\mathbf{k}},\mathbf{1})
\end{array}$$

"dual" description of N=(0,2) Fermi multiplets: needed to introduce subtle interactions

- U(k) anomalies cancel.
- Potentials? (correct symmetries & moduli space)
- 2 standard superpotentials associated with a Fermi multiplet: e.g. at k=1,

$$J_{\check{\lambda}}=\phi\tilde{q}\ ,\quad E_{\check{\lambda}}=0$$

- New D-term-like interactions: super-partners include chiral constraints

$$\mathcal{L}_{\rm int}=\int d^2\theta\left(\alpha q^\dagger\hat{V}\phi+\beta\epsilon^{ijk}q_i\phi_j v\tilde{q}_k^\dagger\right)+c.c.$$

$$\begin{aligned}
\mathcal{L}_{\rm int} &= \int d^2\theta F(\Phi,\bar{\Phi})\hat{V}+c.c. \\
&= \hat{D}F-\sqrt{2}i\bar{\mu}\frac{\partial F}{\partial\bar{\phi}_i}\bar{\psi}_i+\sqrt{2}i\frac{\partial F}{\partial\phi_i}\psi_i\lambda-i\hat{A}_-\left(\frac{\partial F}{\partial\phi_i}D_+\phi_i-\frac{\partial F}{\partial\bar{\phi}_i}D_+\bar{\phi}_i+\frac{\partial^2 F}{\partial\phi_i\partial\bar{\phi}_j}\psi_i\bar{\psi}_j\right)+c.c.
\end{aligned}$$

- Chiral constraints are hard in Lagrangian: presumably break manifest Lorentz symmetry
- Constraints are easy to impose in UV (weakly coupled): basically like chiral bosons

The moduli space, NLSM & UV completion

- Solving the zero potential condition, we find

$$V(\phi_{\mathrm{ADHM}}, \phi_{\mathrm{others}}) = V_1(\phi_{ADHM}) + V_2(\phi_{\mathrm{others}}, \phi_{\mathrm{ADHM}})$$

- Extra fields = 0. The ADHM fields satisfy

$$D^I \equiv q_{\dot\alpha}(\tau^I)^{\dot\alpha}{}_{\dot\beta}\bar{q}^{\dot\beta} + (\tau^I)^{\dot\alpha}{}_{\dot\beta}[a_{\alpha\dot\alpha}, a^{\alpha\dot\beta}] = 0$$

- The moduli space becomes hyper-Kahler: supports (0,4) enhancement of NLSM in IR

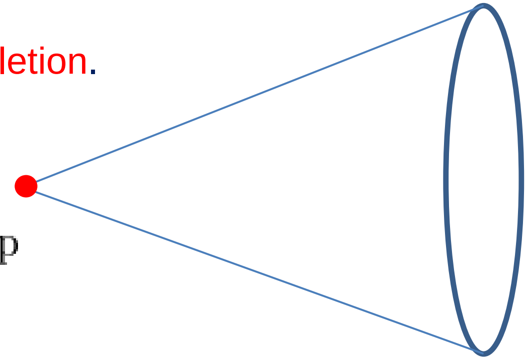
- Non-linear sigma model in IR: (expect from instanton's moduli space approximation)

$$S_{\mathrm{2d}} = \int d^2x \left[- g_{MN}(X)\partial_\mu X^M \partial^\mu X^N + \cdots \right]$$

coordinates of $4c_2k = 12k$ dimensional instanton moduli space

- Moduli space is singular: small instanton singularity. NLSM is incomplete.
- Deep in the Higgs branch, NLSM description shouldn't change.
- Other fields: extra d.o.f. localized at small instanton singularity. UV completion.

ϕ_{extra} are massless only at the tip



Other observables

- elliptic genus: (w/ chemical potentials ~ mass parameters which lift noncompact moduli space)

$$Z_k(\tau, \epsilon_{1,2}, m_a) = \mathrm{Tr}\left[(-1)^F e^{2\pi i\tau H_+} e^{2\pi i\bar{\tau} H_-} e^{2\pi i\epsilon_1(J_1+J_R)} e^{2\pi i\epsilon_2(J_2+J_R)} \cdot \prod_{a\in\mathrm{flavor}} e^{2\pi i m_a F_a}\right]$$

$$H_\pm \equiv \frac{H\pm P}{2} \qquad H_- \sim \{Q, \overline{Q}\}$$

- Easily computable if we have a UV gauge theory. [Benini, Eager, Hori, Tachikawa]

- For simplicity, let us consider single string: (similar works done for higher k's)

$$Z_1^{SU(3)}(v, \epsilon_{1,2}) = -\frac{\eta^2}{\theta_1(\epsilon_{1,2})} \sum_{i=1}^{3} \frac{\eta^4\theta_1(4\epsilon_+ - 2v_i)\theta_1(v_i)}{\prod_{j(\neq i)}\theta_1(v_{ij})\theta_1(2\epsilon_+ - v_{ij})\theta_1(2\epsilon_+ + v_j)}$$

- $v_{i=1,2,3}$: SU(3) chemical potentials
- $\epsilon_{1,2}$: chemical potentials for SO(4) rotation & $SU(2)_R$

- This is very rich data. One can make highly nontrivial tests of our theory with it.

Relation to topological strings on CY3

- Elliptic genus is defined after compactifying spatial circle.

- T-dualize F-theory on $R^{4,1}$ x S^1 x CY_3 ~ type IIB on $R^{4,1}$ x S^1 x B_4
- M-theory on $R^{4,1}$ x [same CY_3]

- #(self-dual strings) = D3 charge (IIB) ~ D2 charge (IIA) ~ M2 charge (M)
- P (IIB) ~ F1 winding (IIA) ~ M2 wrapped on T^2 fiber

- BPS states counted by elliptic genus = BPS states of wrapped M2-branes on CY_3
- This is basically counted by topological string amplitudes, or more precisely by the "Gopakumar-Vafa invariants" at strong coupling limit of topological strings

- We have abundant "experimental data" on the elliptic genus from topological strings.
- Strings of 6d minimal SCFTs: studied in [Haghighat, Klemm, Lockhart, Vafa] 2014

Tests

- Partial data known from topological strings at k=1,2,3 [Haghighat, Klemm, Lockart, Vafa]

$$\log Z(\tau,\epsilon_+,\epsilon_+,\mu) = \sum_{g\geq 0, n\geq 0} (\epsilon_1\epsilon_2)^{g-1}(\epsilon_1+\epsilon_2)^n F_{g,n}(\tau,\mu)$$

- E.g. at k=1, our gauge theory yields

$$\begin{aligned} F_{0,0} &= -\left[\frac{\theta_1(2v_1)\theta_1(v_1)}{\theta_1(v_{12})^2\theta_1(v_{13})^2\theta_1(v_2)\theta_1(v_3)} + (1,2,3\to 2,3,1) + (1,2,3\to 3,1,2)\right] \\ &= e^{-\pi i\tau + 2\pi i v_{12} + 2\pi i v_{23}} \sum_{d_0,d_1,d_2=0}^{\infty} N_{d_0,d_1,d_2}\left(\frac{e^{2\pi i\tau}}{e^{2\pi i v_{12}}e^{2\pi i v_{23}}}\right)^{d_0} e^{2\pi d_1 v_{12}} e^{2\pi d_2 v_{23}} \end{aligned}$$

$d_1 \setminus d_2$	0	1	2	3	4	5
0	1	3	5	7	9	11
1	3	4	8	12	16	20
2	5	8	9	15	21	27
3	7	12	15	16	24	32
4	9	16	21	24	25	35
5	11	20	27	32	35	36

Table 1: q^0

$d_1 \setminus d_2$	0	1	2	3	4	5
0	3	4	8	12	16	20
1	4	16	36	60	84	108
2	8	36	56	96	144	192
3	12	60	96	120	180	252
4	16	84	144	180	208	288
5	20	108	192	252	288	320

Table 2: q^1

$d_1 \setminus d_2$	0	1	2	3	4	5
0	5	8	9	15	21	27
1	8	36	56	96	144	192
2	9	56	149	288	465	651
3	15	96	288	456	735	1080
4	21	144	465	735	954	1371
5	27	192	651	1080	1371	**1632**

Table 3: q^2

$d_1 \setminus d_2$	0	1	2	3	4	5
0	7	12	15	16	24	32
1	12	60	96	120	180	252
2	15	96	288	456	735	1080
3	16	120	456	1012	1788	**2796**
4	24	180	735	1788	**2823**	**4356**
5	32	252	1080	**2796**	**4356**	**5760**

Table 4: q^3

black numbers: computed from topological strings

red: our prediction

complete agreement in black numbers

UnHiggsing SU(3)

- 6d Higgsings are reflected in 2d QFT as massive deformations
- Allowed unHiggsing sequences: all exceptional ~ "terminates after finite sequence"

$$(SU(3)) \leftarrow (G_2, n_{\mathbf{7}} = 1) \leftarrow (SO(7), n_{\mathbf{7}} = 0, n_{\mathbf{8}} = 2) \leftarrow (SO(8), n_{\mathbf{8}_v} = n_{\mathbf{8}_s} = n_{\mathbf{8}_c} = 1)$$

$$\leftarrow \left\{ \begin{array}{l} (SO(N), n_{\mathbf{N}} = N-7, n_{\mathbf{S}} = \cdots)_{N=9,\cdots,12} \\ (F_4, n_{\mathbf{26}} = 2) \leftarrow (E_6, n_{\mathbf{27}} = 3) \leftarrow (E_7, n_{\frac{1}{2}\mathbf{56}} = 5) \leftarrow (E_8, n_{\text{inst}} = 9) \end{array} \right.$$

- Progress w/ G_2 instantons, and SO(7) w/ matters in spinor rep. [work in progress]
- Exceptional instantons' elliptic genera & 1d Witten indices: e.g. G_2 single instanton in 1d

$$Z_1^{G_2} = \frac{t^{\frac{3}{2}}(1+t)(1+t\chi_{\mathbf{7}}^{G_2}(v)+t^2)}{\prod_{i<j}(1-te^{-v_{ij}})(1-te^{v_{ij}})} = t^{\frac{3}{2}} \sum_{n=0}^{\infty} \chi_{(0,n)}^{G_2}(v) t^n$$

[Cremonesi,Ferlito,Hanany,Mekareeya]

$$\oint \frac{d\phi}{2\pi i} \frac{2\sinh\epsilon_+ \cdot 2\sinh\phi \cdot 2\sinh(\epsilon_+ - \phi) \cdot 2\sinh\frac{\epsilon_+ + \phi}{2}}{2\sinh\frac{\epsilon_+ \pm (u - v_{1,2,3})}{2} \cdot 2\sinh\frac{\epsilon_+ - \phi - v_{1,2,3}}{2}} \cdot \frac{1}{2\sinh\frac{\epsilon_+ \pm \phi}{2}}$$

$$= \sum_{i=1}^{3} \frac{2\sinh(2\epsilon_+ - v_i) \cdot 2\sinh\frac{v_i}{2}}{\prod_{j(\neq i)} 2\sinh\frac{v_{ij}}{2} \cdot 2\sinh\frac{2\epsilon_+ - v_{ij}}{2} \cdot 2\sinh\frac{2\epsilon_+ + v_j}{2}} \cdot \frac{1}{2\sinh\frac{v_i}{2} \cdot 2\sinh\frac{2\epsilon_+ - v_i}{2}}$$

one G_2 instanton partition function
from 1d gauge theory

Concluding remarks

- 6d CFTs are hard. Even the 2d QFTs on solitons are hard for many 6d theories.

- We are getting solid results on 2d gauge theories on self-dual strings:
- related to exceptional instantons' ADHM-like descriptions
- Extensions to all exceptional instantons?

  (new ideas: using N=(0,2) UV completions, new gauge theory interactions, …)

- New "viewpoints / computable observables / techniques" for higher dimensional SCFTs under constructions: posing modern challenges to quantum field theorists

- Probing isolated SCFTs via compactifications:
- 6d (2,0) on Riemann surface Σ_2 ~ M5's wrapping Σ_2 : 4d N=2 SCFTs [Gaiotto] [AGT] ……
- D3 + 7-branes wrapping S^2's (w/ 7-brane punctures): 2d N=(0,4) CFTs for self-dual strings
- Related to 4d exceptional SCFTs via compactification to 2d.